IBO

IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICS
HIGHER LEVEL
PAPER 1

Name

Number

Friday 8 November 2002 (afternoon)

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your name and candidate number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Calculator

Make	Model

EXAMINER	TEAM LEADER	IBCA
TOTAL /120	TOTAL /120	TOTAL /120

*Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive **no** marks.*

1. When the polynomial $x^4 + ax + 3$ is divided by $(x-1)$, the remainder is 8. Find the value of *a*.

Working:

Answer:

2. The graph of the function $f(x) = 2x^3 - 3x^2 + x + 1$ is translated to its image, $g(x)$, by the vector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Write $g(x)$ in the form $g(x) = ax^3 + bx^2 + cx + d$.

Working:

Answer:

3. Find the coefficient of x^3 in the binomial expansion of $\left(1-\dfrac{1}{2}x\right)^8$.

Working:

Answer:

4. Find the equations of all the asymptotes of the graph of $y=\dfrac{x^2-5x-4}{x^2-5x+4}$.

Working:

Answers:

5. An integer is chosen at random from the first one thousand positive integers. Find the probability that the integer chosen is

(a)   a multiple of 4;

(b)   a multiple of **both** 4 and 6.

Working:

Answers:

(a) _____

(b) _____

6. Find $\displaystyle\sum_{r=1}^{50} \ln(2^r)$, giving the answer in the form $a \ln 2$, where $a \in \mathbb{Q}$.

Working:

Answer:

_____

7. The functions $f(x)$ and $g(x)$ are given by $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + x$.

The function $(f \circ g)(x)$ is defined for $x \in \mathbb{R}$, **except** for the interval $]a, b[$.

(a) Calculate the value of a and of b.

(b) Find the range of $f \circ g$.

Working:

Answers:

(a) _____

(b) _____

8. Consider the six numbers, 2, 3, 6, 9, a and b. The mean of the numbers is 6 and the variance is 10. Find the value of a and of b, if $a < b$.

Working:

Answers:

_____

_____

Turn over

9. Solve the inequality $x^2 - 4 + \dfrac{3}{x} < 0$.

Working:

Answers:

10. Find an equation for the line of intersection of the following two planes.

$$x + 2y - 3z = 2$$
$$2x + 3y - 5z = 3$$

Working:

Answer:

11. A particle moves in a straight line with velocity, in metres per second, at time t seconds, given by

$$v(t) = 6t^2 - 6t, \ t \geq 0$$

Calculate the total distance travelled by the particle in the first two seconds of motion.

Working:

Answer:

12. Triangle ABC has AB = 8 cm, BC = 6 cm and $B\hat{A}C = 20°$. Find the smallest possible area of $\triangle ABC$.

Working:

Answer:

13. Find $\int (\theta \cos\theta - \theta) \, d\theta$.

Working:

Answer:

14. Find the *x*-coordinate of the point of inflexion on the graph of $y = xe^x$, $-3 \le x \le 1$.

Working:

Answer:

15. The probability density function $f(x)$, of a continuous random variable X is defined by

$$f(x) = \begin{cases} \dfrac{1}{4}x(4-x^2), & 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the **median** value of X.

Working:

Answer:

16. Air is pumped into a spherical ball which expands at a rate of $8\,\text{cm}^3$ per second $(8\,\text{cm}^3\text{s}^{-1})$. Find the **exact** rate of increase of the radius of the ball when the radius is 2 cm.

Working:

Answer:

17. The point $B(a,b)$ is on the curve $f(x) = x^2$ such that B is the point which is closest to $A(6,0)$. Calculate the value of a.

Working:

Answer:

18. Given two non-zero vectors a and b such that $|a+b| = |a-b|$, find the value of $a \cdot b$.

Working:

Answer:

19. The transformation **M** represents a reflection in the line $y = x\sqrt{3}$. The transformation **R** represents a rotation through $\dfrac{\pi}{6}$ radians anticlockwise about the origin. Give a full geometric description of the single transformation which is equivalent to **M** followed by **R**.

Working:

Answer:

20. The tangent to the curve $y = f(x)$ at the point $P(x, y)$ meets the *x*-axis at $Q(x-1, 0)$. The curve meets the *y*-axis at $R(0, 2)$. Find the equation of the curve.

Working:

Answer: